FORMULARIO DI TRIGONOMETRIA

Relazioni fondamentali:

1. $\sin^2(\alpha)+\cos^2(\alpha) = 1 \quad \forall\, a \in \mathbb{R}$
2. $\tan(\alpha) = \dfrac{\sin(\alpha)}{\cos(\alpha)} \quad \forall\, a \neq \dfrac{\pi}{2}+k\pi$
3. $\cot(\alpha) = \dfrac{\cos(\alpha)}{\sin(\alpha)} \quad \forall\, a \neq k\pi$
4. $\cot(\alpha) = \dfrac{1}{\tan(\alpha)} \quad \forall\, a \neq k\dfrac{\pi}{2}$

Archi opposti ed esplementari
(simmetrici rispetto all'asse x)

$\sin(-\alpha) = -\sin(\alpha)$

$\cos(-\alpha) = \cos(\alpha)$

$\tan(-\alpha) = -\tan(\alpha)$

$\cot(-\alpha) = -\cot(\alpha)$

Archi che differiscono di 2π
(congruenti)

$\sin(2\pi+\alpha) = \sin(\alpha)$

$\cos(2\pi+\alpha) = \cos(\alpha)$

$\tan(2\pi+\alpha) = \tan(\alpha)$

$\cot(2\pi+\alpha) = \cot(\alpha)$

Archi supplementari
(simmetrici rispetto all'asse y)

$\sin(\pi-\alpha) = \sin(\alpha)$

$\cos(\pi-\alpha) = -\cos(\alpha)$

$\tan(\pi-\alpha) = -\tan(\alpha)$

$\cot(\pi-\alpha) = -\cot(\alpha)$

Archi che differiscono di π
(simmetrici rispetto all'origine, ossia ruotati di 180°)

$\sin(\pi+\alpha) = -\sin(\alpha)$

$\cos(\pi+\alpha) = -\cos(\alpha)$

$\tan(\pi+\alpha) = \tan(\alpha)$

$\cot(\pi+\alpha) = \cot(\alpha)$

Archi complementari
(simmetrici rispetto alla bisettrice del I e III quadrante)

$\sin\left(\dfrac{\pi}{2}-\alpha\right) = \cos(\alpha)$

$\cos\left(\dfrac{\pi}{2}-\alpha\right) = \sin(\alpha)$

$\tan\left(\dfrac{\pi}{2}-\alpha\right) = \cot(\alpha)$

$\cot\left(\dfrac{\pi}{2}-\alpha\right) = \tan(\alpha)$

Archi che differiscono di π/2
(ruotati di 90°)

$\sin\left(\dfrac{\pi}{2}+\alpha\right) = \cos(\alpha)$

$\cos\left(\dfrac{\pi}{2}+\alpha\right) = -\sin(\alpha)$

$\tan\left(\dfrac{\pi}{2}+\alpha\right) = -\cot(\alpha)$

$\cot\left(\dfrac{\pi}{2}+\alpha\right) = -\tan(\alpha)$

Archi la cui somma è 3π/2
(simmetrici rispetto alla bisettrice del II e IV quadrante)

$\sin\left(\dfrac{3}{2}\pi-\alpha\right) = -\cos(\alpha)$

$\cos\left(\dfrac{3}{2}\pi-\alpha\right) = -\sin(\alpha)$

$\tan\left(\dfrac{3}{2}\pi-\alpha\right) = \cot(\alpha)$

$\cot\left(\dfrac{3}{2}\pi-\alpha\right) = \tan(\alpha)$

Archi che differiscono di 3π/2
(ruotati di –90°)

$\sin\left(\dfrac{3}{2}\pi+\alpha\right) = -\cos(\alpha)$

$\cos\left(\dfrac{3}{2}\pi+\alpha\right) = \sin(\alpha)$

$\tan\left(\dfrac{3}{2}\pi+\alpha\right) = -\cot(\alpha)$

$\cot\left(\dfrac{3}{2}\pi+\alpha\right) = -\tan(\alpha)$

FORMULARIO DI TRIGONOMETRIA

Formule di addizione

$$\sin(\alpha+\beta) = \sin(\alpha)\cdot\cos(\beta)+\cos(\alpha)\cdot\sin(\beta)$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cdot\cos(\beta)-\sin(\alpha)\cdot\sin(\beta)$$

$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\cdot\tan(\beta)}$$

$$\cot(\alpha+\beta) = \frac{\cot(\alpha)\cdot\cot(\beta)-1}{\cot(\alpha)+\cot(\beta)}$$

Formule di sottrazione

$$\sin(\alpha-\beta) = \sin(\alpha)\cdot\cos(\beta)-\cos(\alpha)\cdot\sin(\beta)$$

$$\cos(\alpha-\beta) = \cos(\alpha)\cdot\cos(\beta)+\sin(\alpha)\cdot\sin(\beta)$$

$$\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\cdot\tan(\beta)}$$

$$\cot(\alpha+\beta) = \frac{\cot(\alpha)\cdot\cot(\beta)-1}{\cot(\alpha)+\cot(\beta)}$$

Formule di duplicazione

$$\sin(2\alpha) = 2\cdot\sin(\alpha)\cdot\cos(\beta)$$

$$\cos(2\alpha) = \cos^2(\alpha)-\sin^2(\alpha)$$

$$\tan(2\alpha) = \frac{2\cdot\tan(\alpha)}{1-\tan^2(\alpha)}$$

$$\cot(\alpha+\beta) = \frac{\cot^2(\alpha)-1}{2\cdot\cot(\alpha)}$$

Formule di bisezione

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin(\alpha)}{1+\cos(\alpha)}$$

$$\cot\left(\frac{\alpha}{2}\right) = \frac{\sin(\alpha)}{1-\cos(\alpha)}$$

Formule di Werner

$$\sin(\alpha)\cdot\sin(\beta) = \frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))$$

$$\sin(\alpha)\cdot\cos(\beta) = \frac{1}{2}(\sin(\alpha-\beta)-\sin(\alpha+\beta))$$

$$\cos(\alpha)\cdot\cos(\beta) = \frac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta))$$

Formule di Prostaferesi

$$\sin(\alpha)+\sin(\beta) = 2\cdot\sin(\lambda)\cdot\cos(\mu)$$

$$\cos(\alpha)+\cos(\beta) = 2\cdot\cos(\lambda)\cdot\cos(\mu)$$

$$\sin(\alpha)-\sin(\beta) = 2\cdot\cos(\lambda)\cdot\sin(\mu)$$

$$\cos(\alpha)-\cos(\beta) = 2\cdot\sin(\lambda)\cdot\sin(\mu)$$

essendo $\lambda = \frac{\alpha+\beta}{2}$ e $\mu = \frac{\alpha-\beta}{2}$

$$\tan(\alpha)+\tan(\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha)\cdot\cos(\beta)}$$

$$\tan(\alpha)-\tan(\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha)\cdot\cos(\beta)}$$

$$\cot(\alpha)+\cot(\beta) = \frac{\sin(\alpha+\beta)}{\sin(\alpha)\cdot\sin(\beta)}$$

$$\cot(\alpha)-\cot(\beta) = \frac{-\sin(\alpha-\beta)}{\sin(\alpha)\cdot\sin(\beta)}$$

Formule Parametriche

$$\sin(\alpha) = \frac{2t}{1+t^2} \qquad \cos(\alpha) = \frac{1-t^2}{1+t^2} \qquad \tan(\alpha) = \frac{2t}{1-t^2} \qquad \tan(\alpha) = \frac{1-t^2}{2t}$$

essendo $t = \tan\left(\frac{\alpha}{2}\right)$